LETTERS TO THE EDITOR.

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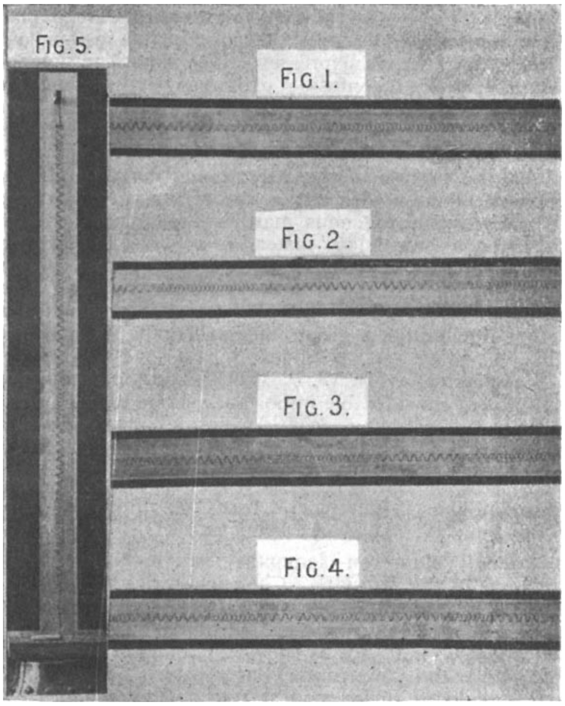
A Simple Model for Demonstrating Beat.

THE phenomenon of beat produced by the interference of two series of waves having nearly the same wave-length can be objectively represented by a model of simple construction.

A spiral, whose diameter and pitch are respectively 2 cm. and 2·5 cm., is made of a steel wire about 1 mm. thick and hung vertically before a white screen. At a distance of a few metres we observe a very regular series of transverse waves. Another spring of exactly the same dimensions is suspended in front of the first spring so as to coincide with each other when they are seen at a distance. If one of the springs is then slightly stretched, there results a small difference in wave-length of the two sets of waves, thus causing them to strengthen in one place and destroy in the other. The distance between these two places becomes less as the difference of wave-length increases. Figs. 1, 2, 3, 4 are the photographs of the springs suspended in the manner just mentioned, and show successive stages of interference produced by stretching the length of the second spring. The result of interference of two such waves evidently corresponds to the phenomenon of beat.



For practical purposes, two springs are suspended from a vertical board, one in front of the other, as shown in Fig. 5. Both ends of the first spring are fixed, while the upper end of the second is likewise fixed and the lower end pulled downwards by means of a string passing through a hook attached to the stand. Standing at a distance in front of the springs we can gradually stretch the second spring by pulling the string and easily observe the corresponding stages of interference in its different phases.

K. HONDA.

Physical Laboratory, Imp. Univ., Tokio, Japan, May 26.

Polar Exploration.

THE following sentence is extracted from an admirable notice of the Arctic explorer, the late Baron Nordenskiöld, by Prof. A. G. Nathorst, himself renowned as an investigator of the Polar regions:—

"It may perhaps be of interest at the present time to recall the fact that we in Sweden have always taken for granted that the leader of a scientific expedition must be a naturalist, to whom the commander of the vessel has to be subordinate."—*Geograph. Anzeiger*, ii. p. 129, September, 1901.

The marked success of the Swedish expeditions, not merely in pure science, but also in geographical discovery and the safe return of their members, gives this opinion weight as well as interest. Of course it is not what the president of the Geographical Section of the British Association calls "the good old British plan."

CIVILIAN.

ON THE CLUSTERING OF GRAVITATIONAL MATTER IN ANY PART OF THE UNIVERSE.

IN the Mathematical and Physical Science Section of the British Association, Lord Kelvin delivered a discourse on "The Absolute Amount of Gravitational Matter in any large Volume of Interstellar Space." Gravitational matter, according to our ideas of universal gravitation, would be all matter. Now was there any matter which was not subject to the law of gravitation? He thought he might say with absolute decision that there was. They were all convinced, with their President, that ether was matter, but they were forced to say that the properties of molar matter were not to be looked for in ether as generally known to them by action resulting from force between atoms and matter, ether and ether, and atoms of matter and ether. Here he was illogical when he said between matter and ether, as if ether were not matter. It was to avoid an illogical phraseology that he used the title "gravitational matter." Many years ago he had given strong reason to feel certain that ether was outside the law of gravitation. They need not absolutely exclude, as an idea, the possibility of there being a portion of space occupied by ether beyond which there was absolute vacuum—no ether and no matter. They admitted that that was something that one could think of; but he did not believe any living scientific man considered it in the slightest degree probable that there was space surrounding our universe beyond which there was no ether and no matter. Well, if ether went through all space, then it was certain that ether could not be subject to the law of mutual gravitation between its parts, because if it were subject to mutual attraction between its parts its equilibrium would be unstable, unless it were infinitely incompressible. But here again he was reminded of the critical character of the ground on which they stood in speaking of or giving very definite propositions beyond what they saw or felt by experiment. He was afraid he must here express a view different from that which Prof. Rücker announced in his address, when he said that continuity of matter implied absolute resistance to condensation. They had no right to bar condensation as a property of ether. While admitting ether not to have any atomic structure, it was postulated as a material which performed functions of which they knew something, and which might have properties allowing it to perform other functions of which they were not yet cognisant. If they considered ether to be matter, they postulated that it had rigidity enough for the vibration of light, but they had no right to say that it was absolutely incompressible. They must admit that sufficiently great pressure all round could condense the ether in a given space, allowing the ether in surrounding space to come in towards the ideal shrinking surface. When he said that ether might be outside the law of gravitation he assumed that it was not infinitely incompressible. He admitted that if it were infinitely incompressible, then it might be subject to the law of mutual gravitation between its parts; but to his mind it seemed infinitely improbable that ether was infinitely incompressible, and it appeared more consistent with the analogies of the known properties of molar matter, which should be their guides, to suppose that ether had not the quality of exerting an infinitely great force against compressing action of gravitation. Hence if they assume that it extended through all space, ether must be outside the law of gravitation, that is to say, truly imponderable. He remembered the contempt and self-complacent compassion with which sixty years ago he himself, he was afraid and most of the teachers of that time looked upon the ideas of the elderly people who went before them, who spoke of "the imponderables." He feared that in this, as in a great many other things in science, they had to hark back to the dark ages of fifty, sixty or a hundred years ago, and that they must admit there was something which they could not refuse to call matter, but which was not subject to the Newtonian law of gravitation. That the sun, stars, planets, and meteoric stones were all of them ponderable matter was true, but the title of his paper implied that there was something else. Ether was not any part of the subject of his paper; what he dealt with was gravitational matter, ponderable matter. Ether they relegated, not to a limbo of imponderables, but to distinct species of matter which had inertia, rigidity, elasticity, compressibility, but not heaviness. In a paper he had already published he had given strong reasons for limiting to a definite amount the quantity of matter in space known to astronomers. He could scarcely avoid using the word "universe," but he meant our universe, which might be a very small affair after all, occupying a very small portion of all the space in which there is ponderable matter.

Supposing a sphere of radius $3\cdot09.10^{16}$ kilometres (being the distance at which a star must be to have parallax $0''\cdot001$) to have within it, uniformly distributed through it, a quantity of matter equal to one thousand million times the sun's mass, the velocity acquired by a body placed originally at rest at the surface would, in five million years, be about 20 kilometres per second, and in twenty-five million years would be 108 kilometres per second (if the acceleration remained sensibly constant for so long a time). Hence if the thousand million suns had been given at rest twenty-five million years ago, uniformly distributed throughout the supposed sphere, many of them would now have velocities of twenty or thirty kilometres per second, while some would have less and some probably greater velocities than 108 kilometres per second; or, if they had been given thousands of million years ago at rest so distributed that now they were equably spaced throughout the supposed sphere, their mean velocity would now be about 50 kilometres per second (*Phil. Mag.*, August 1901, pp. 169, 170). This is not unlike the measured velocities of stars, and hence it seems probable that there might be as much matter as one thousand million suns within the distance $3\cdot09.10^{16}$ kilometres. The same reasoning shows that ten thousand million suns in the same sphere would produce velocities far greater than the known star velocities, and hence there is probably much less than ten thousand million times the sun's mass in the sphere considered. A general theorem discovered by Green seventy-three years ago regarding force at a surface of any shape, due to matter (gravitational, or ideal electric, or ideal magnetic) acting according to the Newtonian law of the inverse square of the distance, shows that a non-uniform distribution of the same total quantity of matter would give greater velocities than would the uniform distribution. Hence we cannot, by any non-uniform distribution of matter within the supposed sphere of $3\cdot09.10^{16}$ kilometres radius, escape from the conclusion limiting the total amount of the matter within it to something like one thousand million times the sun's mass.

Lord Kelvin then went on to compare the sunlight with the light from the thousand million stars, each being supposed to be of the same size and brightness as our sun; and stated that the ratio of the apparent brightness of the star-lit sky to the brightness of our sun's disc would be $3\cdot87.10^{-13}$. This ratio (*Phil. Mag.*, August 1901, p. 175) varies directly with the radius of the containing sphere, the number of equal globes per equal volume being supposed constant; and hence to make the sum of the apparent area of discs 3·87 per cent. of the whole sky, the radius must be $3\cdot09.10^{27}$ kilometres. With this radius light would take $3\frac{1}{4}.10^{14}$ years to travel from the outlying stars to the centre. Irrefragable dynamics proves that the life of our sun as a luminary is probably between 50 and 100 million years; but to be liberal, suppose each of our stars to have a life of 100 million years as a luminary, and it is found that the time taken by light to travel from the outlying stars to the centre of the sphere is three and a quarter million times the life of a star. Hence it follows that to make the whole sky aglow with the light of all the stars at the same time the commencements of the stars must be timed earlier and earlier for the more and more distant ones, so that the time of the arrival of the light of every one of them at the earth may fall within the durations of the lights of all the others at the earth. His supposition as to uniform density is quite arbitrary; but nevertheless he thought it highly improbable that there could be enough stars (bright or dark) to make a total of star-disc-area more than 10^{-12} or 10^{-11} of the whole sky.

To help to understand the density of the supposed distribution of 1000 million suns in a sphere of $3\cdot09.10^{16}$ kilometres radius, imagine them arranged exactly in cubic order, and the volume per sun is found to be $123\cdot5.10^{39}$ cubic kilometres, and the distance from one star to any one of its six nearest neighbours would be $4\cdot98.10^{13}$ kilometres. The sun seen at this distance would probably be seen as a star of between the first and second magnitude; but supposing our 1000 million suns to be all of such brightness as to be stars of the first magnitude at distance corresponding to parallax $1''\cdot0$, the brightness at distance $3\cdot09.10^{16}$ kilometres would be one one-millionth of this; and the most distant of our stars would be seen through powerful telescopes as stars of the sixteenth magnitude. Newcomb estimated from 30 to 50 million as the number of stars visible in modern telescopes. Young estimated at 100 million the number visible through the Lick telescope. This larger estimate is only one-tenth of our assumed thousand million masses equal to the sun, of which, however, nine hundred million might be either non-luminous, or, though luminous, too distant to be seen by us at their actual distances from the earth. Remark also that it is only for facility of counting that we have reckoned our universe as a thousand million suns; and that the meaning of our reckoning is that the total amount of matter within a sphere of $3\cdot09.10^{16}$ kilometres radius is a thousand million times the sun's mass. The sun's mass is $1\cdot99.10^{27}$ metric tons, or $1\cdot99.10^{33}$ grammes. Hence our reckoning of our supposed spherical universe is that the ponderable part of it amounts to $1\cdot99.10^{42}$ grammes, or that its average density is $1\cdot61.10^{-23}$ of the density of water.

Lord Kelvin returned to the question of sum of apparent areas, the ratio of which to 4π, the total apparent area of the sky viewed in all directions, is given by the formula (*Phil. Mag.*, August 1901, p. 175) $a = \frac{3N}{4}\left(\frac{a}{r}\right)^2$; provided its amount is so small a fraction of unity that its diminution by eclipses, total or partial, may be neglected. In this formula, N is a number of globes of radius a uniformly distributed within a spherical surface of radius r. For the same quantity of matter in N′ globes of the same density, uniformly distributed through the same sphere of radius r, we have $\frac{N'}{N} = \left(\frac{a}{a'}\right)^3$, and therefore $\frac{a'}{a} = \frac{a}{a'}$. With $N = 10^9$, $r = 3\cdot09\ 10^{16}$ kms.;

and a (the sun's radius) $=7.10^5$ kms.; we had $a=3{\cdot}87.10^{-13}$. Hence $a'=7$ kms. gives $a'=3{\cdot}87.10^{-8}$; and $a''=1$cm. gives $a''=1/36{\cdot}9$. Hence if the whole mass of our supposed universe were reduced to globules of density 1·4 (being the sun's mean density), and of 2 cms. diameter, distributed uniformly through a sphere of $3{\cdot}09.10^{16}$ kms. radius, an eye at the centre of this sphere would lose only 1/36·9 of the light of a luminary outside it! The smallness of this loss is easily understood when we consider that there is only one globule of 2 cms. diameter per 360,000,000 cubic kilometres of space, in our supposed universe reduced to globules of 2 cms. diameter. Contrast with the total eclipse of the sun by a natural cloud of water spherules, or by the cloud of smoke from the funnel of a steamer.

Let now all the matter in our supposed universe be reduced to atoms (literally brought back to its probable earliest condition). Through a sphere of radius r let atoms be distributed uniformly in respect to gravitational quality. It is to be understood that the condition "uniformly" is fulfilled if equivoluminal globular or cubic portions, small in comparison with the whole sphere, but large enough to contain large numbers of the atoms, contain equal total masses, reckoned gravitationally, whether the atoms themselves are of equal or unequal masses, or of similar or dissimilar chemical qualities. As long as this condition is fulfilled, each atom experiences very approximately the same force as if the whole matter were infinitely fine-grained, that is to say, utterly homogeneous.

Let us therefore begin with a uniform sphere of matter of density ρ, gravitational reckoning, with no mutual forces except gravitation between its parts, given with every part at rest at the initial instant; and let it be required to find the subsequent motion. Imagining the whole divided into infinitely thin concentric spherical shells, we see that every one of them falls inwards, as if attracted by the whole mass within it collected at the centre. Hence our problem is reduced to the well-known students' exercise of finding the rectilinear motion of a particle attracted according the inverse square of the distance from a fixed point. Let x_0 be the initial distance, $\frac{4\pi\rho}{3}x_0{}^3$ the attracting mass, v and x the velocity and distance from the centre at time t. The solution of the problem, for the time during which the particle is falling towards the centre is

$$\tfrac{1}{2}v^2=\frac{4\pi\rho}{3}x_0{}^3\left(\frac{1}{x}-\frac{1}{x_0}\right),$$

and

$$t=\sqrt{\frac{3}{8\pi\rho}}\left(\frac{\pi}{2}-\theta+\tfrac{1}{2}\sin 2\theta\right)=\frac{\pi}{2}\sqrt{\frac{3}{8\pi\rho}}\left[1-\frac{2\theta}{\pi}\left(1-\frac{\sin 2\theta}{2\theta}\right)\right],$$

where θ denotes the acute angle whose sine is $\sqrt{\frac{x}{x_0}}$. This shows that the time of falling through any proportion of the initial distance is the same whatever be the initial distance; and that the time (which we shall denote by T) of falling to the centre is $\tfrac{1}{2}\pi\sqrt{\frac{3}{8\pi\rho}}$. Hence in our problem of homogeneous gravitational matter given at rest within a spherical surface and left to fall inwards, the augmenting density remains homogeneous; and the time of shrinkage to any stated proportion of the initial radius is inversely as the square root of the density.

To apply this result to the supposed spherical universe of radius $3{\cdot}09.10^{16}$ kilometres, and mass equal to a thousand million times the mass of our sun, we find the gravitational attraction on a body at its surface gives acceleration of $1{\cdot}37.10^{-13}$ kms. per second per second This therefore is the value of $\frac{4\pi\rho}{3}x_0$, with one second as the unit of time and one kilometre as the unit of distance; and we find $\mathrm{T}=52{\cdot}8.10^{13}$ seconds $=16{\cdot}8$ million years. Thus our formulas become

$$\tfrac{1}{2}v^2=1{\cdot}37.10^{-13}\,x_0\left(\frac{x_0}{x}-1\right),$$

giving

$$v=5{\cdot}23.10^{-7}\sqrt{x_0\left(\frac{x_0}{x}-1\right)}$$

and

$$t=52{\cdot}8.10^{13}\left[1-\frac{2\theta}{\pi}\left(1-\frac{\sin 2\theta}{2\theta}\right)\right];$$

whence, when $\sin\theta$ is very small,

$$t=52{\cdot}8.10^{13}\left(1-\frac{4\theta^3}{3\pi}\right).$$

Let now for example $x_0=3{\cdot}09.10^{16}$ kms., and $\frac{x_0}{x}=10^7$; and therefore $\sin\theta=\theta=3{\cdot}16.10^{-4}$; whence $v=291{,}000$ kms. per second, and $t=\mathrm{T}-7080$ seconds $=\mathrm{T}-2$ hours approximately.

By these results it is most interesting to know that our supposed sphere of perfectly compressible fluid, beginning at rest with density $1{\cdot}61.10^{-23}$ of that of water, and of any magnitude large or small, and left unclogged by ether to shrink under the influence of mutual gravitation of its parts, would take nearly seventeen million years to reach ·0161 of the density of water, and about two hours longer to shrink to infinite density at its centre. It is interesting also to know that if the initial radius is $3{\cdot}09.10^{16}$ kilometres the inward velocity of the surface is 291,000 kilometres per second at the instant when its radius is $3{\cdot}09.10^9$ and its density ·0161 of that of water. If now, instead of an ideal compressible fluid, we go back to atoms of ordinary matter of all kinds as the primitive occupants of our sphere of $3{\cdot}09.10^{16}$ kms. radius, all these conclusions, provided all the velocities are less than the velocity of light, would still hold; notwithstanding the ether occupying the space through which the atoms move. This would, I believe,[1] exercise no resistance whatever to uniform motion of an atom through it; but it would certainly add quasi inertia to the intrinsic Newtonian inertia of the atom itself moving through ideal space void of ether; which, according to the Newtonian law, would be exactly in proportion to the amount of its gravitational quality. The additional quasi inertia must be exceedingly small in comparison with the Newtonian inertia, as is demonstrated by the Newtonian proofs, including that founded on Kepler's laws for the groups of atoms constituting the planets, and movable bodies experimented on at the earth's surface.

In one thousand seconds of time after the density ·0161 of the density of water is reached, the inward surface velocity would be 305,000 kilometres per second, or greater than the velocity of light; and the whole surface of our condensing globe of gas or vapour or crowd of atoms would begin to glow, shedding light inwards and outwards. All this is absolutely realistic except the assumption of uniform distribution through a sphere of the enormous radius of $3{\cdot}09.10^{16}$ kilometres, which we adopted temporarily for purposes of illustration. The enormously great velocity (291,000 kms. per second) and rate of acceleration (13·7 kms. per second per second) of the boundary inwards, which we found at the instant of density ·0161 of that of water, are due to greatness of the primitive radius and the uniformity of density in the primitive distribution.

To come to reality according to the most probable judgment present knowledge allows us to form, suppose at many millions, or thousands of millions, or millions of millions of years ago, all the matter in the universe to have been atoms very nearly at rest[1] or quite at rest; more densely distributed in some places than in others, of infinitely small average density through the whole of infinite space. In regions where the density was then greater than in neighbouring regions, the density would become greater still; in places of less density, the density will become less; and large regions will quickly become void or nearly void of atoms. These large void regions would extend so as to completely surround regions of greater density. In some part or parts of each cluster of atoms thus isolated, condensation would go on by motions in all directions not generally convergent to points, and with no perceptible mutual influence between the atoms until the density becomes something like 10^{-6} of our ordinary atmospheric density, when mutual influence by collisions would begin to become practically effective. Each collision would give rise to a train of waves in ether. These waves would carry away energy, spreading it out through the void ether of infinite space. The loss of energy thus taken away from the atoms would reduce large condensing clusters to the condition of gas in equilibrium[2] under the influence of its own gravity only, or rotating like our sun or moving at moderate speeds as in spiral nebulas, &c. Gravitational condensation would at first produce rise of temperature, followed later by cooling and ultimately freezing, giving solid bodies, collisions between which will produce meteoric stones such as we see them. We cannot regard as probable that these lumps of broken-looking solid matter (something like the broken stones used on our macadamised roads) are primitive forms in which matter was created. Hence we are forced in this twentieth century to views regarding the atomic origin of all things closely resembling those presented by Democritus, Epicurus, and their majestic Roman poetic expositor, Lucretius.

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[1] "On the Motion produced in an Infinite Elastic Solid by the Motion through the Space occupied by it of a Body acting on it only by Attraction or Repulsion." Cong. International de Physique, Paris, Vol. of Report. (*Phil. Mag.* August, 1900).

THE CHEMISTRY OF THE CYGNIAN STARS AND BASIC ROCKS.

I HAVE recently received from Prof. Suess the following important letter, with a request that it should be sent for publication in NATURE. It is obvious that Prof. Suess' striking generalisation will lead to many interesting inquiries; for my part I shall lose no time in making the additional researches for which he asks. The results on the chromosphere spectrum obtained during the eclipse of 1898 now being published by the Royal Society will also, I think, throw some light on the question. NORMAN LOCKYER.

Vienna, October 7.

MY DEAR SIR,—In reading your highly suggestive and instructive book on Inorganic Evolution and your last papers in *Proc.* Roy. Soc. and NATURE, I was struck by what you say on the spectrum of α Cygni, and beg leave to submit a question.

I believe I am in accord with the best masters of geology of our time in regarding our earth as a ball of NiFe, surrounded by a silicious slag. This slag has parted (or has differentiated) into two zones, one richer in SiAl and felspathic minerals (trachyte, granite, &c.), and the other richer in SiMg (peridotite and serpentine, olivine-rock, lherzolite, dunite, kyschtymite, &c.), and both extremes are united by a host of intermediate rocks. The SiAl group is lighter, exterior and partly used up in forming sediments, the SiMg group is lower or interior and related by the universal occurrence of traces of Ni, and by other features, to NiFe. They are, in fact, the acid and the basic group of Bunsen, Durocher and all their followers.

Now Prof. Vogt, of Christiania, has in a series of remarkable papers shown that the metallic ores accompanying the SiAl rocks are different from the ores of the SiMg group, and that certain metals are very characteristic for each group, and more especially for certain ultra-basic rocks. The same author has, in *Zeitschrift für prakt. Geolog.*, 1898, p. 326, given a list of the elements characteristic for the ores of each group. This list does not regard the rocks themselves, but only the ores, and I have thought to give a better approach to the sum of occurring elements by introducing a few trifling changes, viz. in noting Si, Al and also Cu on both sides, and adding C, according to South African experience, to the basic list.

(1) Acid rocks: *Si*, *K*, *Li*, Be, *Al*, *W*, *U*, *Ce*, *Y*, Cu, *Sn*, *Zr*, *Th*, B, F.

(2) Basic rocks: Si, *Ca*, Al, Ba, Sr, *Mg*, *Fe*, Mn, *Ni*, *Co*, *Cr*, *Cu*, V, *Ti*, *Pt* and allied metals, *C*, P, *S*, Cl.

(The most characteristic members are italicised.)

The ultra-basic rocks have their exact equivalent in the meteoric stone of Chassigny.

The great number of analyses of meteor irons by Cohen recently published by the Berlin Academy gives Fe, Ni, Co, Cr, Cu, C, P, S, Cl, joined in single cases by SiO_2, MgO and CaO. You know that Davison also found Pt and probably Ir. It is Vogt's list of metals from basic rocks.

Ångström's older list of Fraunhofer lines gives H and Na, and besides these Ca, Ba, Mg, Fe, Mn, Ni, Co, Cr and Ti.

These are all elements accompanying the basic rocks, and although your own later observations show the existence of lines of some of the characteristic metals of the acid series, such as K, Li, U and Ce, I must conclude that Vogt's basic list is more distinctly represented in the sun's absorbing layer.

Now your comparison of the sun's chromosphere with the enhanced lines of α Cygni gives a quite similar list for α Cygni: Mg, Ca, Fe, Ti, Mn, Ni, Cr, V, Cu, Sr, and the impression is that the metallic vapours, which accompanied the intrusion of ultra-basic rocks and sometimes, as in Norway, gathered as ores at the circumference of these ultra-basic intrusive rocks, must have been of a remarkable likeness to those of the sun's chromosphere and of α Cygni.

One might even be induced to go a step further. Among the ores of the ultra-basic rocks Vogt distinguishes two varieties, which he calls the oxydic, characterised by the prevalent occurrence of Ti, and the sulfidic ores (nickeliferous pyrrhotite). And I find that you remark γ Cygni to be distinguished by the prevalence of Ti, α Cygni by Fe, Cr and Ni, and β Orionis by Si and Mg—corresponding to these two varieties of ores and to the intrusive rock SiMg (β Orionis being the country rock of the ores). But I am very far from proposing any conclusions whatever in this imperfect state of knowledge, and only venture to point out the interest which is attached to the examination of a number of elements named by you on p. 58, "Inorganic Evolution"—which have only been investigated by you with lower dispersion, and which embrace several typical representatives of the acid series—and to the special research of metals like W, U, Ce, Sn and others of this series.

The question, which I take leave to submit, is: Have we indeed to suppose that metallic vapours answering to metals from acid rocks are less represented in the sun and stars than those from basic rocks, or is it some secondary cause, the nature of their spectrum or other, which gives the present seeming prevalence to the metals from basic rocks?

I beg you, dear sir, to accept the expression of my highest esteem.

Yours most respectfully,
EDW. SUESS.

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[1] "On Mechanical Antecedents of Motion, Heat and Light"; Brit. Assoc. Rep., part ii. 1854; *Edin. New Phil. Jour.* i. 1855; *Comptes rendus*, xl. 1855; Kelvin's "Collected Math. and Phys Papers," vol. ii. art. lxix.

[2] Homer Lane, *American Journal of Science*, 1870, p. 57; Sir W. Thomson, *Phil. Mag.*, March 1887, p. 287.